A Spatiotemporal-chaos-based Encryption Having Overall Properties Considerably Better Than Advanced Encryption Standard

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(February 8, 2008)

Abstract

Spatiotemporal chaos of a two-dimensional one-way coupled map lattice is used for chaotic cryptography. The chaotic outputs of many space units are used for encryption simultaneously. This system shows satisfactory cryptographic properties of high security; fast encryption (decryption) speed; and robustness against noise disturbances in communication channel. The overall features of this spatiotemporal chaos based cryptosystem are better than chaotic cryptosystems known so far, and also than currently used conventional cryptosystems, such as the Advanced Encryption Standard (AES).

PACS number(s): 05.45.Vx, 05.45.Ra, 43.72.+q

In the last ten years of 20th century, secure communication by utilizing chaos synchronization had attracted much attention due to the hope that the random-like behavior of chaos and the sensitivity of chaotic trajectories to initial conditions may provide new cryptographic methods for hiding private information [1–6]. However, the recent development of chaos communication is rather disappointing. Various cryptoanalyses have exposed some inherent drawbacks of chaotic cryptosystems, such as low security even with chaotic dynamics completely hidden [7–10], slow algorithms [7], and weakness in resistance against noise disturbances in the transmission channels, which make it difficult to promote the chaos communication into practical service.

On the other hand, conventional cryptography has become a modern science since the work of Shannon [11], and recently some standard and effective cryptographic methods have been suggested, e.g., for the symmetrical scheme Data Encryption Standard (DES) and Advanced Encryption Standard (AES issued in 2000). In particular, the AES algorithm has been shown to have a satisfactory trade-off in the overall properties of security, performance, and software and hardware realizations, having no counterpart in chaotic cryptosystems until now [7,8,12].

In our recent paper [13] we have shown that a one-dimensional (1D) coupled map lattice may reach very high practical security for chaotic cryptography, which has in the same time fairly fast encryption and relatively short synchronization time in comparison with other currently known chaos-based encryption methods. However, the overall properties of the system of [13] are still not as good as AES: its encryption speed is slower (2 to 3 times) than AES, and its avalanche effect is larger (3 to 4 times) than AES (i.e., its robustness against channel noise is weaker than AES). In order that a chaos-based encryption method can stand up as a new and useful cryptographic technique, it must show its significant advantages over conventional secure communication methods, and this is the task of the present communication.

In this paper, we propose a 2D coupled map lattice for spatiotemporal chaotic cryptography. The most significant point is that we are able to use a large number of chaotic

sequences generated by different sites to encode (decode) plaintexts simultaneously [4], and thus the efficiency of cryptography can be greatly enhanced. Moreover, we can choose proper coupling structure and encryption transformation and separate the functions of driving bits and pure cipher bits for reaching high security together with satisfactory robustness against channel noise. The overall properties of our cryptosystem can be thus incomparably better than the chaotic cryptosystems known so far, and be also considerably better than the most effective conventional cryptographic methods, like AES.

We use the following chaotic coupled map lattice for transmitter encryption,

$$\begin{aligned} x_n(0) &= \frac{S_n(N/2, N/2)}{2^\nu}, \quad \text{for even } N \\ x_{n+1}(j) &= (1-\varepsilon) f_j[x_n(j)] + \varepsilon f_{j-1}[x_n(j-1)], \\ f_0(x) &= 4x(1-x) \\ f_j(x) &= a_j x(1-x), \quad j = 1, 2, ..., m \end{aligned} \tag{1a}$$

$$\begin{aligned} z_{n+1}(0,0) &= (1-\varepsilon) f[z_n(0,0)] + \varepsilon f_m[x_n(m)] \\ y_n(0,0) &= z_n(0,0) \times 2^h \mod 1 \\ f(x) &= 4x(1-x) \end{aligned} \tag{1b}$$

$$\begin{aligned} y_{n+1}(1,0) &= (1-\varepsilon) f[y_n(1,0)] + \varepsilon f[y_n(0,0)] \\ y_{n+1}(0,1) &= (1-\varepsilon) f[y_n(0,1)] + \varepsilon f[y_n(0,0)] \\ y_{n+1}(j_1,0) &= (1-\varepsilon) f[y_n(j_1,0)] + \varepsilon\{0.8 f[y_n(j_1-1,0) + 0.2 f[y_n(j_1-2,0)]\}, \quad j_1 = 2, ..., N \quad (1c) \\ y_{n+1}(0,j_2) &= (1-\varepsilon) f[y_n(0,j_2)] + \varepsilon\{0.2 f[y_n(0,j_2-1) + 0.8 f[y_n(0,j_2-2)]\}, \quad j_2 = 2, ..., N \\ y_{n+1}(j_1,j_2) &= (1-\varepsilon) f[y_n(j_1,j_2)] + \varepsilon\{0.5 f[y_n(j_1-1,j_2) \ + 0.5 f[y_n(j_1,j_2-1)]\}, \quad (1d) \\ & 2 \leqslant j_1 + j_2 \leqslant N \end{aligned}$$

$$\begin{aligned} S_n(j_1,j_2) &= [K_n(j_1,j_2) + I_n(j_1,j_2)] \mod 2^\nu, \\ K_n(j_1,j_2) &= [\mathrm{int}(y_n(j_1,j_2) \times 2^\mu] \mod 2^\nu, \quad 2 \leqslant j_1 + j_2 \leqslant N \end{aligned} \tag{1e}$$

where I_n's are the plaintexts assumed to be private, $S_n(j_1, j_2)$ are the ciphertexts transmitted in the open channel, and $\mathbf{a} = (a_1, a_2, ..., a_m)$ are the adjustable control parameters, serving as the secret key. For specific we will, throughout the paper, take parameters $h = 26$, $\mu = 52$, $\nu = 32$, $\varepsilon = 0.99$, $m = 3$, $N = 6$, and $a_1 = a_2 = a_3 = 3.9$.

The schematic encoding structure is given in Fig.1. First, we use a 1D lattice chain of length $m = 3$, for setting the secret keys $\mathbf{a} = (a_1, a_2, a_3)$ in Eq.(1a). This arrangement can guarantee full sensitivity of the encryption processes to all key parameters on one hand, and reduce as much as possible the number of noncipher sites on the other hand. Second, following the 1D chain is a 2D one-way coupled map lattice of Eqs.(1c) and (1d), producing $M = 25$ chaotic sequences (see the square sites of Fig.1), in parallel for fast encryption. The 2D structure is for reducing the system length and thus effectively reducing synchronization time and the associated error bit avalanche. And the neighbor and next to the neighbor coupling structure of Eq.(1c) is for enormously increasing the cost of any inverse analytical computation attacks. Third, there are two mod operations Eqs.(1b) and (1e) which may considerably enhance the sensitivity of chaos synchronization to the key parameter variations, and thus reach high practical encryption security [14]. Finally, from the M outputs sequences $S_n(j_1, j_2)$, a single arbitrarily chosen sequence [$S_n(3, 3)$ in Fig.1] is separated from the remaining cipher sequences as the driving sequences in Eq.(1a) and Fig.1, and this separation will be shown later of great significance in strengthening the robustness of the system against channel noise. With the feed back of the driving in Eq.(1a) the encoding system turns to be high-dimensional nonautonomous spatiotemporal chaos.

The decryption transformation of the receiver can be obtained symmetrically by replacing $x_n(j)$, $z_n(0, 0)$, $y_n(j_1, j_2)$, the key $\mathbf{a} = (a_1, a_2, ..., a_m)$, with $x'_n(j)$, $z'_n(0, 0)$, $y'_n(j_1, j_2)$, and the test key $\mathbf{b} = (b_1, b_2, ..., b_m)$, respectively. Only one among the $M = 25$ transmitted signals, $x'_n(0) = S_n(3, 3)/2^\nu$, serves as the driving signal for the spatiotemporal chaos synchronization of the receiver; and decoding operations become

$$I'_n(j_1, j_2) = [S_n(j_1, j_2) - K'_n(j_1, j_2)] \mod 2^\nu, \tag{2}$$

$$K_n'(j_1,j_2) = [\mathrm{int}(y_n'(j_1,j_2)\times 2^{\mu}] \mod 2^{\nu}$$

$I_n'$are the received plaintexts. By setting $\mathbf{b} = \mathbf{a}$, the receiver can realize synchronization to the chaotic transmitter, and correctly extract the message as

$$y_n'(j_1,j_2) = y_n(j_1,j_2), \quad K_n'(j_1,j_2) = K_n(j_1,j_2), \tag{3}$$
$$I_n'(j_1,j_2) = I_n(j_1,j_2),\ 2 \leqslant j_1 + j_2 \leqslant N$$

Now, let us evaluate our cryptosystem in the aspects of performance, security and robustness. The first significant point of Eqs.(1) and Eq.(2) is that we fully take the advantage of spatiotemporal chaos in the performance. A large number of chaotic sites in the lattice network can be used for encryption (decryption) in parallel. For the present parameters, we have totally 31 coupled maps, among which 25 chaotic sites can produce ciphertexts simultaneously. Thus, the encryption (decryption) efficiency is extremely high. Specially, we can produce 350-Mbit ciphers per second with our 750MHz CPU computer. In comparison, for conventional block ciphers, AES can have encryption speed of 96-Mbit, 80-Mbit, and 66-Mbit ciphers per second for 128-bit, 192-bit, and 256-bit key sizes (for a 600 MHz CPU PC) [12], respectively, and for conventional stream ciphers a 32-bit linear feedback shift register (LFSR), which has very low security, has encryption speed of 20-Mbits per second with our same PC.

A crucial problem for the scheme of Fig.1 is whether $M = 25$ keystreams can be effectively used in parallel. A positive answer can be available only if different keystreams are practically uncorrelated. In Figs.2(a) and 2(b) we plot the mutual correlation $C_{24,23}(\tau)$ (for the definition, see Eq.(3.2) of [4]) and mutual information $I(K_n(2,4); K_n(2,3)) = H(K_n(2,4)) - H(K_n(2,4) \mid K_n(2,3))$ between two neighbor keystreams $K_n(2,4)$ and $K_n(2,3)$, with $H(K_n(2,4))$ and $H(K_n(2,4) \mid K_n(2,3))$ being information entropy and conditional entropy, respectively. The behaviors of Fig.2 are not changed if we take any other pais of sites. Thus, all the keystreams produced by different sites are practically independent from each other, and can be satisfactorily used for the parallel encryption (decryption) purpose.

Second, we evaluate the practical security of the cryptosystem, i.e., the resistance of the chaotic system against the plaintext-known and public-structure attacks, by applying the error function analysis (EFA) [13]. The error function $e(\mathbf{b})$ can be computed with the available information and an arbitrary test key $\mathbf{b}$ as

$$e_{j_1,j_2}(\mathbf{b}) = \frac{1}{T}\sum_{n=1}^{T}\left|i_n'(j_1,j_2) - i_n(j_1,j_2)\right|, \tag{4}$$

$$i_n(j_1,j_2) = \frac{I_n(j_1,j_2)}{2^{32}}, \qquad i_n'(j_1,j_2) = \frac{I_n'(j_1,j_2)}{2^{32}}$$

By varying $\mathbf{b}$ any third party may find the basin of $e_{j_1,j_2}(\mathbf{b})$ and locate the key position $\mathbf{b} = \mathbf{a}$ by identifying the minimum error $e(\mathbf{b} = \mathbf{a}) = 0$, and then illegally decode any future plaintexts.

In Figs.3(a)-(c) we fix $b_2 = b_3 = 3.9$ and plot $e(b_1)$ vs b_1. $e(b_1)$ has a basin around its minimum $e(b_1 = a_1 = 3.9) = 0$, and the basin is extremely narrow, showing the high sensitivity of $e(b_1)$ to key b_1. Away from the basin $e(b_1)$ has a flat distribution fluctuating around its average value. In Fig.3(d) we plot $e(b_1, b_2)$ in b_1-b_2 plane with $b_3 = a_3 = 3.9$, an extremely small $e(b_1, b_2)$ basin hole is located in the 2D parameter space. The a_j parameter region available for spatiotemporal chaos is at least in $a_j = [3.6, 4.0]$, $j = 1, 2, 3$. By applying the analysis similar to [13] the volume of the key basin can be estimated from Fig.3 as $V < (10^{-12})^3 \approx 10^{-36}$, and the probability to find the key basin by an arbitrary test is $P < 0.4^{-3} \times 10^{-36} \approx 10^{-35}$. From the flat distribution of $e(b)$ in Figs.3(a), (b), (d) and from the numerous local minima of $e(b)$ in Fig.3(c) one can hardly find any adaptive approach to reveal the $e(b)$ basin more effective than random tests, and the third party has to make at least 10^{35} tests to find the key basin. This can be done by the currently best computer in the world at least for 10^{16} years (the estimation can be seen in [13]).

Apart from the EFA method, there are many other known attack methods in both conventional and chaotic cryptographies, such as linear and differential attacks [15], nonlinear dynamic forecasting attacks [9,10], and so on. All these methods are based on predicting plaintexts by revealing the plaintext-induced statistical changes of ciphertexts. We have

computed probability distributions of the ciphertexts generated at different sites for various plaintexts. It is clearly observed that for essentially different plaintexts the probabilities of ciphertexts have the same uniform distributions, and they are not distinguishable from each other. Therefore, the above attacks can be hardly practical. Analytic inverse computation can also break the security of Eq.(1). A simple evaluation shows that for our system this inverse computation requires cost much more than the EFA method.

Our cryptosystem is better than AES with security in the following two aspects. First, with secure key well hidden our system is practically one-time pad cipher, while AES is definitely not either with hidden key. Second, our system can easily increase its security level, e.g., increasing m by one we can increase the security factor by 10^{12} with almost no encryption speed reduction. For AES there exists a maximum security level (2^{128} for given plaintext attack). For further increasing security from this maximum level one has to increase the block length, and then considerably decrease the encryption speed. Therefore, our system can much easier in resistance against stronger attacks developed by new powerful computers, like possible quantum computers.

The last point to be emphasized is that the cryptographic structure of Eq.(1) makes the secure communication robust against channel noise disturbances. All self-synchronizing cryptosystems have a disadvantage of error avalanche, i.e., one bit error in ciphertext may cause a large number of error bits in the received plaintext due to the finite synchronization recovering time, and slower synchronization may cause more serious avalanche effect. In this regard, our system has several useful advantages. First, our decoding system takes a strong coupling $(1-\varepsilon) = 10^{-2} << 1$ which yields rather small largest Lyapunov exponent, leading to quickly damping of any desynchronous disturbances in the receiver. This guarantees short synchronization time (26 iterations for our parameter combination) and relatively small error avalanche (17 iterations). Second, chaos synchronization between the transmitter and the receiver can be realized by using a single driving sequence. Thus, among all $M =$ 25 transmitted ciphertext sequences, only one sequence is used for driving, i.e., only $\frac{1}{25}$ transmitted bits has avalanche problem, and other bit errors do not cause avalanche effect.

Therefore, in average the avalanche destruction can be considerably reduced. Third and the most important, in order to reduce error avalanche people commonly include some additional bits for protection of driving signal, that increases the costs of both cryptography and signal transmission. In doing so our system has a great advantage of low cost over AES because in the latter case one should protect all transmitted bits (each has equal avalanche effect) while for the former only the driving signal, i.e., $\frac{1}{M}$ of the total transmitted bits, has the avalanche effect and needs to be particularly protected.

The cryptosystem of Eqs.(1) and (2) with the given parameters has been realized in a software experiment for duplex voice transmission by using local university campus network. The experimental set is given by Fig.2 of [13] with cryptosystem replaced by Eqs.(1) and (2) of this paper. Experimental dialogue can be performed stably between two phones with standard voice quality and standard speaking speed for arbitrarily long time. Moreover, we have realized also experimental duplex voice transmission by using normal city phone line where channel noise is considerably larger than that of network. In this experiment our system works perfectly well with high security (high sensitivity to the key parameter change) and satisfactory robustness against noise. A detailed discussion on this experimental set and a detailed experimental comparison between our system and AES will be reported soon.

This work was supported by the National Natural Science Foundation of China under 10175010 and Nonlinear Science Project.

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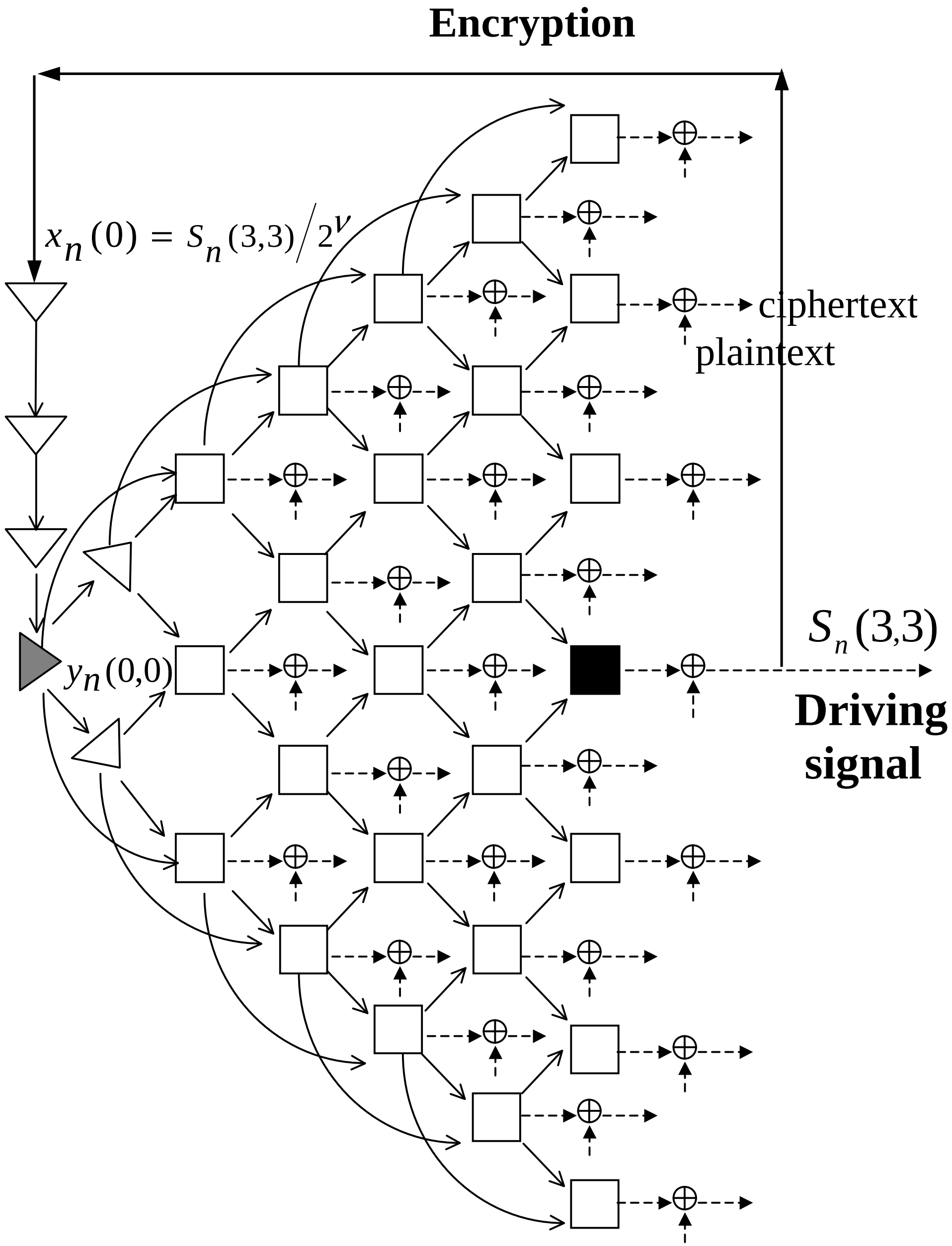
Captions of Figures

Fig.1 A schematic figure of the cryptosystem Eqs.(1) for $N = 6, m = 3$. All the $M = 25$ square sites produce ciphers each 32 bits for an iteration while the other six triangle sites not. All the driving and nondriving ciphers are transmitted in a same open channel.

Fig.2(a) and (b) The mutual correlation $C_{24,23}(\tau)$ [Eq.(3.2) in [4]] and the mutual information $I(K_n(2,4); K_n(2,3))$ between $K_n(2,4)$ and $K_n(2,3)$ vs time distance τ and sample number G, respectively. The behaviors are not changed if we take any other pairs of keystreams.

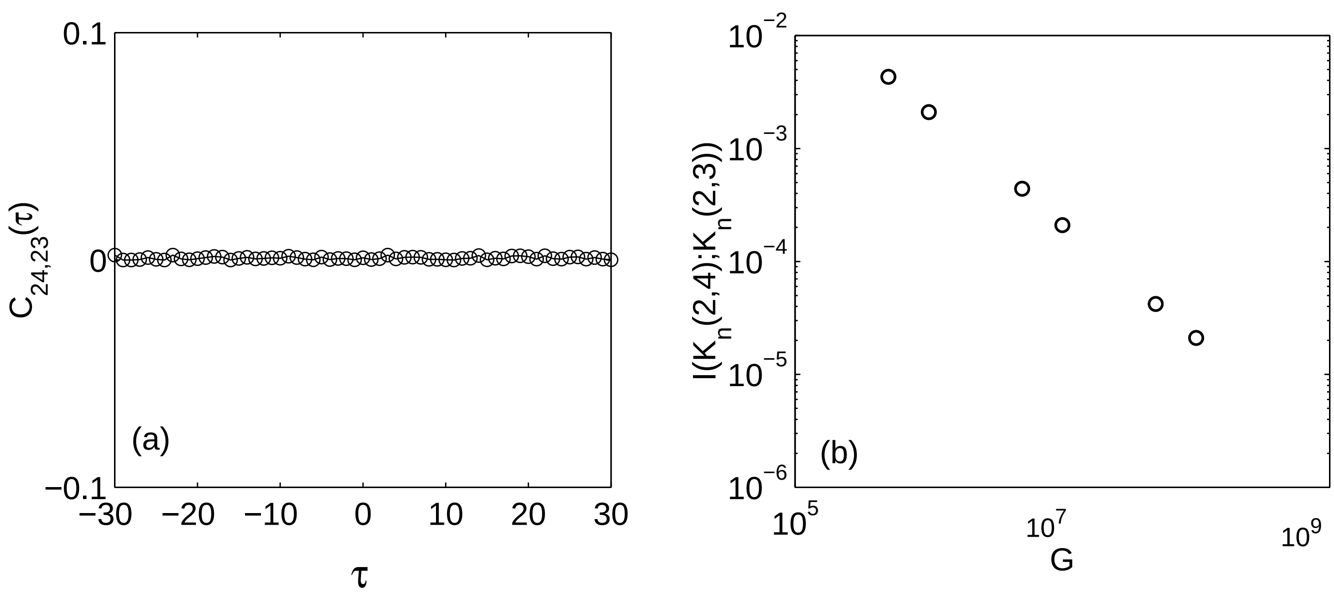
Fig.3 (a)-(c) Error function $e(b_1)$ given in Eq.(4) plotted vs b_1, $b_2 = b_3 = 3.9$. (d) $e(b_1, b_2)$ plotted in b_1–b_2 plane, $b_3 = 3.9$.

Fig.1



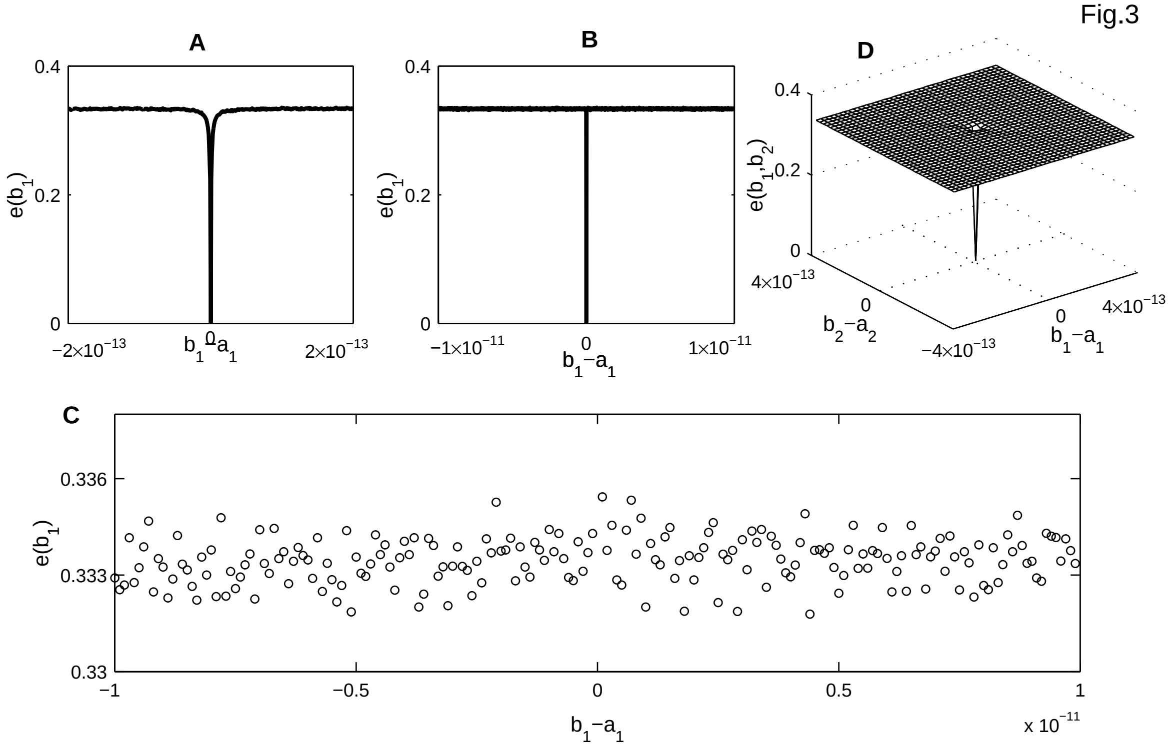
Encryption
$x_n(0) = S_n(3,3) / 2^{\nu}$
ciphertext
plaintext
$S_n(3,3)$
$y_n(0,0)$
Driving signal

Fig.2



0.1
0
−0.1
−30
−20
−10
0
10
20
30
$C_{24,23}(\tau)$
τ
(a)
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{5}
10^{7}
10^{9}
$I(K_n(2,4);K_n(2,3))$
G
(b)

Fig.3



A
0.4
0.2
0
e(b1)
−2×10−13
b1−a1
2×10−13
B
e(b1)
−1×10−11
1×10−11
D
e(b1,b2)
4×10−13
b2−a2
−4×10−13
C
0.336
0.333
0.33
−1
−0.5
0.5
1
x 10−11